Long-term stable microwave signal extraction from mode-locked lasers

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Abstract: Long-term synchronization between two 10.225 GHz microwave signals at +10 dBm power level, locked to a 44.26 MHz repetition rate passively mode-locked fiber laser, is demonstrated using balanced optical-microwave phase detectors. The out-of-loop measurement result shows 12.8 fs relative timing jitter integrated from 10 Hz to 10 MHz. Long-term timing drift measurement shows 48 fs maximum deviation over one hour, mainly limited by drift of the out-of-loop characterization setup itself. To the best of our knowledge, this is the first time to demonstrate long-term (>1 hour) 3 mrad-level phase stability of a 10.225 GHz microwave signal extracted from a mode-locked laser.

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OCIS codes: (320.7160) Ultrafast technology; (120.3940) Metrology; (120.5050) Phase measurement; (350.4010) Microwaves.

References and links

#82703 - $15.00 USD Received 7 May 2007; revised 13 Jun 2007; accepted 27 Jun 2007; published 5 Jul 2007
1. Introduction

Mode-locked lasers show a great potential to generate ultralow-jitter microwave signals encoded in its pulse repetition frequency. However, it is a highly nontrivial task to transfer the low noise properties of the pulse train in the optical domain to the electronic domain, and extract a drift-free, ultralow-jitter microwave signal with a power level sufficient for the intended application from an optical pulse train. For distribution and synchronization of microwave signals from optical pulse trains in large-scale, high-precision timing distribution systems [1, 2, 3], it is crucial to convert the optical pulse train into a drift-free, low-jitter microwave signal with a satisfactory power level in a long-term stable way. Readout of microwave signals from atomic optical clocks [4, 5] is another important motivation for long-term drift-free extraction of microwave signals from optical pulse trains.

The extraction of a microwave signal from an optical pulse train using direct photodetection, the most commonly used technique for microwave signal extraction from pulse trains, suffers from excess phase noise [6, 7, 8]. The major origin of this excess noise is amplitude-to-phase conversion in the photodetectors and microwave mixers [8, 9]. The intensity noise and power drifts of optical pulse trains can be converted into a significant amount of excess timing jitter and drifts, and degrade the long-term stability of the extracted microwave signals. Nevertheless, careful suppression of these excess noise sources has been used to demonstrate microwave signal extraction from optical pulse trains at 10 GHz with 3.3 fs timing jitter measured from 0.1 Hz to 1 MHz [4] and later even 0.8 fs measured from 1 Hz to 1 MHz at a power level of -15 dBm [5]. Although the short-term jitter could be suppressed down to 1 fs level, the slow phase drift of extracted microwave signals is still the major limitation, for example, ∼56 fs (∼3.5 mrad at 10 GHz) drift over 100 seconds caused by ∼0.3 % amplitude fluctuation as shown in Ref. [4].

To circumvent the amplitude-to-phase conversion and to ensure long-term stable microwave signal extraction directly at the microwave power level needed for subsequent experiments, a balanced optical-microwave phase detector was recently proposed and demonstrated [10]. It is based on the precise phase detection in the optical domain using a differentially-biased Sagnac fiber loop and synchronous detection. Because the phase error between the optical pulse train and the microwave signal is detected by electro-optic sampling in the optical domain, it has a potential to be more robust against power and thermal drifts.

In this paper, we present detailed noise analysis and out-of-loop characterization results over a broad range of time scales of the balanced optical-microwave phase detectors. The out-of-loop performance between two optoelectronic phase-locked loops (PLLs) using two of these balanced optical-microwave phase detectors shows 12.8 fs out-of-loop relative jitter between two 10.225 GHz microwave signals at an output power level of +10 dBm integrated from 10 Hz to 10 MHz. In addition, a separate long-term timing drift measurement shows that the maximum deviation in the extracted microwave signals is less than 48 fs in one hour. This long-term stability is, to the best of our knowledge, the first time to achieve a 3 mrad-level phase stability of a 10.225 GHz microwave signal extracted from a mode-locked laser over an hour time scale. Note that currently the long-term measurement is mainly limited by the drift of the mixer and electronic amplifiers used in the out-of-loop characterization setup, which drifts up to 41 fs (in 1 hour) and 48 fs (in 4 hours) even though the temperature of the characterization setup is actively stabilized.

2. Operation and noise analysis of balanced optical-microwave phase detectors

In this section, we present the operation and noise analysis of balanced optical-microwave phase detectors. Figure 1(a) shows the schematic diagram of the optoelectronic PLL using a balanced optical-microwave phase detector. In this diagram, note that the reference signal
\( f_R/2 \) is generated by detecting the fundamental repetition rate \( f_R \) with a separate photodiode and dividing this frequency by a factor of two with a frequency divider. Figure 1(b) shows the relative phase relationship between the optical pulse train, the voltage-controlled oscillator (VCO) output signal, and the reference signal. Major parameters used in the derivation in this section are labelled in Fig. 1(a) and (b).

\[
\Phi(t) = \Phi_0 \sin(2\pi f_0 t + \theta_e) + \Phi_m \sin(\pi f_R t + \Delta\phi),
\]

where \( P_{\text{avg,in}} \) is the average optical power of the input pulse train to the Sagnac-loop and \( T_R = 1/f_R \) is the period of the pulse train.

The phase modulator in the Sagnac-loop is driven by the sum of the signal from (1) the VCO with a frequency equal to a multiple of repetition rate, \( f_0 = N f_R \) (when locked), and (2) the reference signal with a frequency of half the repetition rate, \( f_r = f_R/2 \) (where \( f_R = 1/T_R \) is the repetition rate of the pulse train):

The optical pulse train applied to the Sagnac-loop can be written as

\[
P_{\text{in}}(t) = P_{\text{avg,in}} T_R \sum_{n=-\infty}^{\infty} \delta(t - nT_R),
\]

Fig. 1. (a) Schematic diagram of the optoelectronic phase-locked loop (PLL) using a balanced optical-microwave phase detector. BPF, bandpass filter; VCO, voltage-controlled oscillator. (b) Relative positions of the optical pulse train (blue pulse train), the VCO output signal (red sinusoidal signal), and the reference signal (grey sinusoidal signal). For illustrative simplicity, \( N \) is set to \( N = 1 \).
where \( \Phi_0 \) is the amplitude of phase modulation from the VCO, \( \theta_e \) is the phase error between the pulse train and the VCO signal, \( \Phi_m \) is the amplitude of phase modulation from the reference signal, and \( \Delta \phi \) is the fixed relative phase between the pulse train and the reference signal.

After circulating in the Sagnac-loop interferometer, the output optical power can be expressed as 
\[
P(t) = (1 - L)P_{\text{avg}}(t) \sin^2(\phi/2),
\]
where \( L \) is the loss in the Sagnac-loop and \( \phi \) is the phase difference between counterpropagating pulses. This equation is expanded as
\[
P(t) = P_{\text{avg}}T_R \sum_{n=-\infty}^{\infty} \sin^2 \left[ \frac{1}{2} \Phi_0 \sin(2\pi f_0 t + \theta_e) + \Phi_m \sin(\pi f_R t + \Delta \phi) \right] \delta(t - nT_R),
\]
where \( P_{\text{avg}} = (1 - L)P_{\text{avg,in}} \) is the average power of the output pulse train from the Sagnac-loop.

By the travelling-wave nature of the phase modulator, the high frequency VCO signal (10.225 GHz in this work) generates a unidirectional phase modulation, i.e., only the copropagating pulse experiences the phase modulation while the counterpropagating pulse does not. Because the same unidirectional property is not applied to the reference signal (22.13 MHz in this work), we place the phase modulator in such a way that counterpropagating pulses experience opposite phases by the reference signal. This is the reason why the modulation depth is multiplied by a factor of two only for the reference signal in Eq. 3.

Now suppose the frequency is locked, that is, \( f_0 = N f_R \). Since the pulse train will sit on the zero-biasing points, i.e., \( t_p = nT_R \) at the locked state, the following approximations are valid: \( \sin(2\pi f_0 t + \theta_e) = \sin(2\pi N n + \theta_e) \simeq \theta_e \) and \( \sin(\pi f_R t + \Delta \phi) = \sin(n\pi + \Delta \phi) \). In addition, \( \sin^2(\phi/2) \simeq 1/4 \) holds for small phase modulations. With these linear approximations, the optical power at the photodiode (Eq. 3) is then expressed as
\[
P(t) = P_{\text{avg}}T_R \sum_{n=-\infty}^{\infty} \left[ \Phi_0^2 \cos^2(\theta_e) + \Phi_m^2 \left( 1 - \cos(2\Delta \phi) \right) \right] \delta(t - nT_R).
\]

By the Fourier transform of Eq. 4, one can extract the amplitude at the reference frequency, \( f_R/2 \). The result is the following:
\[
P \left( \pm \frac{f_R}{2} \right) = 2\pi P_{\text{avg}} \Phi_0 \Phi_m \sin(\Delta \phi) \theta_e \delta \left( f \mp \frac{f_R}{2} \right).
\]

We clearly see the relative phase \( \Delta \phi \) should be set to \( \pi/2 \) to maximize the phase detection sensitivity. In this condition, the pulse train sits exactly on the maxima/minima of the modulation signal as shown in Fig. 1(a). When the pulse train is received by a photodiode with a responsivity of \( R \) (A/W), bandpass-filtered at \( f_R/2 \), and amplified by a transimpedance gain of \( G \) (V/A), the input voltage to the downconversion mixer is
\[
V_2(t) = 2RG \Phi_0 \Phi_m \theta_e \cos(\pi f_R t).
\]

Note that the phase error \( \theta_e \) is encoded in the amplitude of the signal. To convert the phase error signal to the baseband, \( V_2(t) \) is mixed in phase with the reference signal \( V_1(t) \):
\[
V_1(t) = V_1 \sin(\pi f_R t + \Delta \phi) = V_1 \cos(\pi f_R t).
\]

When these two signals are mixed, the baseband voltage is
\[
V_d = \alpha V_1(t)V_2(t)_{|f< f_R/2} = [\alpha RG \Phi_0 \Phi_m V_1] \theta_e,
\]
where \( \alpha \) (V\(^{-1}\)) is the conversion efficiency of the downconversion mixer. This is the phase error signal output from the balanced optical-microwave phase detector. Therefore, the phase
detection sensitivity $K_d$ (V/rad) is

$$K_d = \frac{V_d}{\theta_e} = \alpha RGP_{avg} \Phi_0 \Phi_m V_1.$$  \hspace{1cm} (9)

The fundamental noise limit of this phase detector comes from the shot noise of the photodetection process. This is a valid assumption because the relative-intensity noise (RIN) of passively mode-locked solid-state lasers has a bandwidth limited by the long upper-state lifetime [11], typically less than 1 MHz. At the reference signal ($f_R/2 = 22.13$ MHz in this work) where the synchronous detection is operated, the laser source was confirmed to be shot noise limited.

Here, the phase noise floor from shot noise will be derived. When the system is locked, the average optical power received at the Sagnac-loop output photodiode is

$$\langle P_{\text{locked}} \rangle = \langle P_{\text{avg}} T_R \sum_{n=-\infty}^{\infty} \Phi_m^2 \delta(t-nT_R) \rangle = \Phi_m^2 P_{\text{avg}}.$$  \hspace{1cm} (10)

The shot noise current power spectral density in $A^2/Hz$ unit is

$$\langle i_{\text{shot}}^2 \rangle = 2qI_0 = 2qRG \Phi_m^2 P_{\text{avg}},$$  \hspace{1cm} (11)

where $q$ (C) is the electron charge.

When we suppose the reference signal is noise-free, the voltage noise density from the mixer output at the baseband in $V^2/Hz$ unit is

$$\langle V_{\text{d,shot}}^2 \rangle = \frac{1}{2} \alpha^2 V_1^2 G^2 \langle i_{\text{shot}}^2 \rangle = \alpha^2 V_1^2 qRG \Phi_m^2 P_{\text{avg}}.$$  \hspace{1cm} (12)

From Eqs. 9 and 12, the single-sideband (SSB) phase noise density floor due to shot noise in rad$^2$/Hz at the carrier frequency $Nf_R$ (the frequency of the VCO output signal) is expressed as

$$S_{\varphi,\text{shot}} = \frac{1}{2} \frac{\langle V_{\text{d,shot}}^2 \rangle}{K_d^2} = \frac{\alpha^2 V_1^2 qRG \Phi_m^2 P_{\text{avg}}}{2(\alpha RGP_{avg} \Phi_0 \Phi_m V_1)^2} = \frac{q}{2RP_{\text{avg}} \Phi_0^2}.$$  \hspace{1cm} (13)

This sets the absolute limit in achievable residual jitter. From this relationship, it is clear that higher optical power ($P_{\text{avg}}$) as well as phase modulation depth (RF-power) from the VCO output ($\Phi_0$) enables the minimum residual jitter.

3. Experimental setup

Figure 2 shows the schematic of the experimental setup for out-of-loop relative timing jitter measurements between the two 10.225 GHz microwave signals locked to a free-running 44.26 MHz, 1550 nm stretched-pulse Er-doped fiber mode-locked laser. Two nearly identical optoelectronic PLLs based on balanced optical-microwave phase detectors were built with 10.225 GHz (the 231st harmonic of the fundamental repetition rate, $N = 231$) VCOs (PSI DRO-10.225). The average input optical power to each Sagnac-loop ($P_{\text{avg,in}}$) is 5 mW, and +4 dBm of VCO output power is used to close the PLL. From each VCO, +10 dBm output power at 10.225 GHz can be extracted for external measurements. More detailed information on the experimental implementation of balanced optical-microwave phase detectors can be found in Ref. [10].
To evaluate the out-of-loop relative timing jitter between the two extracted microwave signals, the outputs from the locked VCOs are mixed in quadrature in the out-of-loop phase noise characterization setup. The baseband phase error signal is amplified by a low-noise amplifier (G=10 non-inverting amp with AD797) and monitored by a vector signal analyzer (Agilent 89410A) for phase noise spectral density measurements and a data acquisition system (Agilent 34970A) for long-term drift measurements. Note that only the out-of-loop phase noise characterization setup is actively temperature-stabilized to enable a long-term measurement, while both PLLs are not temperature-stabilized or otherwise shielded against environmental perturbations.

4. Short-term timing jitter measurement results

Figure 3 summarizes the single-sideband (SSB) phase noise spectra for the in-loop and out-of-loop performances between two PLLs when both PLLs are optimized for long-term stable operation. In the locked state, the in-loop timing jitters integrated from 10 Hz to 10 MHz for PLL 1 (curve (b)) and PLL 2 (curve (c)) are 19.2 fs and 18.8 fs, respectively. Most of the noise contribution is from the high frequency (>100 kHz) peak. The noise floor above 5 MHz is caused by the bandpass filter at the reference signal frequency. To evaluate the resolution of the out-of-loop characterization setup, we measured the residual phase noise (curve (d)) when the same 10.225 GHz microwave signal is split and applied to both input ports of the mixer in quadrature. The background timing jitter of the characterization setup itself is 0.8 fs integrated from 10 Hz to 10 MHz. The out-of-loop relative timing jitter between the two extracted 10.225 GHz microwave signals (curve (e)) integrated from 10 Hz to 10 MHz is 12.8 fs. Both in-loop and out-of-loop timing jitters are mostly determined by the high-frequency noise between 100 kHz and 1 MHz. The reason why the in-loop jitter is larger than the out-of-loop jitter is mainly from the high frequency peak characteristic caused by the finite loop bandwidth of about 100 kHz. The enhanced high frequency peak in the in-loop characteristic is reduced in the out-of-loop measurement by limiting the loop filter bandwidths of the PLLs. Although the in-loop results show strong noise suppressions in the low frequency range due to the loop filter integrator, the out-of-loop noise does not show the same strong suppression. In...
addition, the out-of-loop noise level almost did not move even when changing the parameters such as optical power and microwave signal power levels, which should scale the noise level when the system is shot noise limited as discussed in Section 2. With the parameters used in the experiment (photodiode responsivity: \( R = 0.9 \, \text{A/W} \), average output optical power from the Sagnac-loops: \( P_{\text{avg}} = 2 \, \text{mW} \), and phase modulation depth of VCO output: \( \Phi_0 = 0.3 \, \text{rad} \)), the shot-noise limited theoretical SSB phase noise level for each loop should be \( S_{\phi, \text{shot}} = -152 \, \text{dBc/Hz} \) as derived in Eq. 13 (line (f)). The measured out-of-loop noise level is about 30 dB worse than the theoretical shot noise limited level. We believe that we are limited by the uncorrelated noise sources from the reference signal, electronic excess noise from the amplification of the output pulse train, and the limited isolation between ports (LO-RF, LO-IF) of the downconversion mixer. We are currently investigating the sources of this non-scalable noise.

![Fig. 3. Single-sideband (SSB) phase noise spectra at 10.225 GHz from 10 Hz to 10 MHz: (a) free-running VCO (taken from datasheet); (b) in-loop phase noise of PLL 1; (c) in-loop phase noise of PLL 2; (d) residual phase noise of the out-of-loop characterization setup; (e) out-of-loop relative phase noise between PLL 1 and PLL 2; (f) phase noise level in the ideal condition, when both PLLs are shot-noise limited and there is no excess electronic noise sources. The out-of-loop measurement shows 12.8 fs relative jitter between two extracted microwave signals. The in-loop jitters are 19.2 fs and 18.8 fs for PLL 1 and 2, respectively.](image)

5. **Long-term timing drift measurement results**

Although the short-term background jitter (0.8 fs) provides enough resolution for the characterization, the long-term timing drift of the characterization setup itself shows significant amount of drift. Even though the temperature of the characterization setup is actively stabilized within 0.4 °C_{pp} (0.07 °C_{rms}) over 10 hours, at certain time frames, up to 41 fs (in 1 hour) and 48 fs (in 4 hours) timing drifts are observed, as shown in Fig. 4(a). The measured timing drift and the temperature are not clearly correlated, and the exact reasons for this rather abrupt drift are currently not fully understood. Path length variations in the microwave cables and connectors...
Fig. 4. (a) Long-term background timing drift measurement of the characterization setup. Although the temperature is actively stabilized within 0.41 °C (maximum-minimum) over 10 hours, at certain time frames, up to 41 fs (in 1 hour) and 48 fs (in 4 hours) timing drifts are observed. (b) Long-term out-of-loop drift measurement between two locked VCOs shows that the maximum timing deviation is within 48 fs over one hour. The data was taken at every 5 seconds.
that have not been stabilized might be a major cause for this abrupt and large phase fluctuations (note that a 50 fs drift corresponds to a path length change of only 15 µm). This drift in the characterization setup sets the limitation of the long-term drift measurement.

Figure 4(b) shows the result for long-term timing drift measurement between the two 10.225 GHz microwave signals when both VCOs are locked. The output voltage from the characterization setup was recorded every 5 seconds over a time span of one hour. The relative timing between the two microwave signals shows a maximum deviation of 48 fs over one hour. This corresponds to 3 mrad phase stability at the 10.225 GHz carrier frequency. As shown in Fig. 4(a), the characterization setup itself may contribute up to >40 fs drift. Therefore, the long-term timing drift measurement result in Fig. 4(b) is currently limited by the characterization setup itself. To overcome the limitation of the long-term drift measurement, it is desirable to use an optical technique for the timing detection to avoid drift of microwave components, and we are currently working on the new measurement method.

6. Conclusion

In summary, we have demonstrated long-term stable (<3 mrad over 1 hour) microwave signal extraction from a mode-locked laser using balanced optical-microwave phase detectors at the frequency of 10.225 GHz and the power level of +10 dBm. This excellent long-term phase stability is achieved by electro-optic sampling of the microwave signal with the optical pulse train in a Sagnac-loop interferometer. The relative short-term out-of-loop timing jitter integrated from 10 Hz to 10 MHz is 12.8 fs. The measured long-term timing deviation is within 48 fs over one hour, which is mainly limited by the drift of the characterization setup itself. The demonstrated performance is currently limited by the electronic noise sources rather than the shot noise, and the potential of noise scalability is not yet fully exploited. By identifying and removing the non-ideal technical noise sources, we expect to reach shot noise limited performance with long-term drift-free operation.

Acknowledgment

This research was supported in part by the Office of Naval Research under contract ONR-N00014-02-1-0717, the Air Force Office of Scientific Research under contract FA9550-07-1-0014, and the EU Commission in the Sixth Framework Program, Contract No. 011935 EUROFEL. J. Kim acknowledges fellowship support from the Samsung Scholarship Foundation.